Fall 2004 SDM5008 Advanced Control for Robotics

Lecture Note 8: Mujoco Tutorial

Prof. Wei Zhang

Southern University of Science and Technology

zhangw3@sustech.edu.cn
https://www.wzhanglab.site/

Outline

- **Short introduction to Simulation**
- Introduction to Mujoco
- Python Example

- **What is Simulation?** · real world / · virtual (gravity)
  - Real-world physics are often described by functions, ODE or PDE
    - · too expensive to do experiment; static relation; differential equation

· simulation: modeling physical world to do prediction

· circuit simulation

· Electromagnetic simulation — world model

– Finite-element / mechanics

– control system simu — robotic sys simulation

all models are wrong, some are useful

  - All simulators essentially solve the ODEs and/or PDEs corresponding to a physical process of interest

· IMU. static model. sensor: $y_{meas} = \begin{bmatrix} a_x \\ a_y \\ a_z \\ \dot{\alpha} \\ \dot{\beta} \\ \dot{\gamma} \end{bmatrix} = f(x) + \underline{v}$

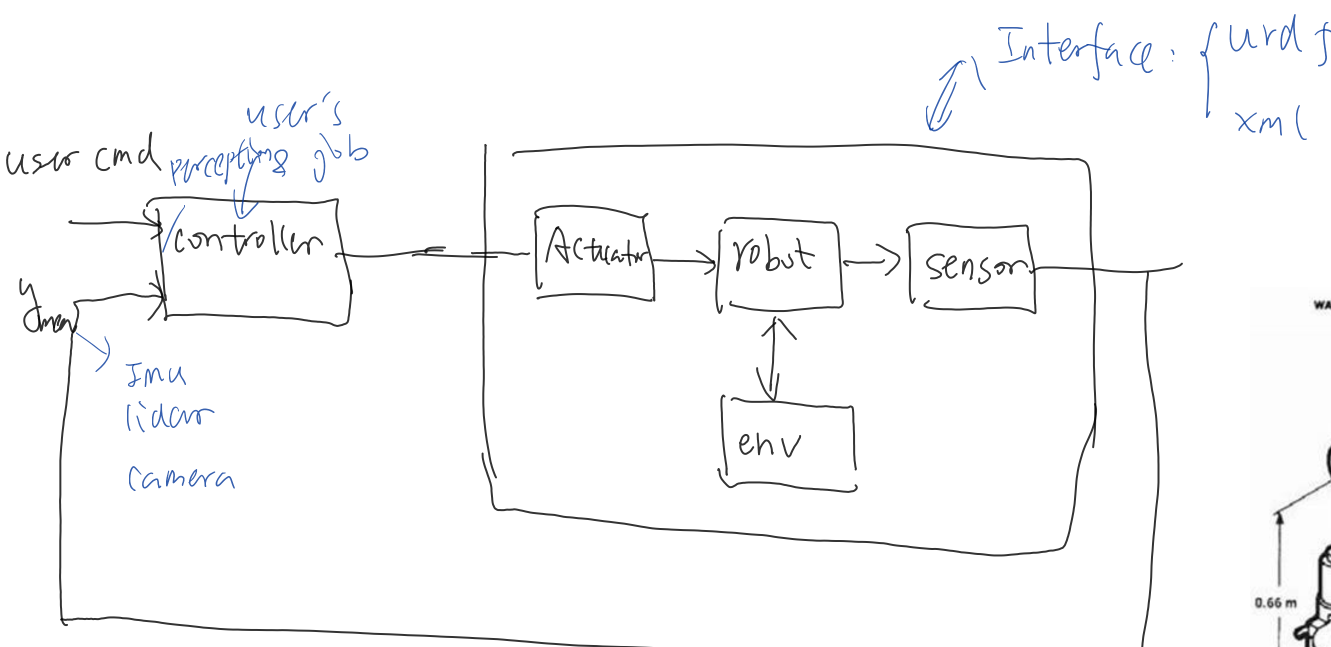
lidar $y = f(x, env)$

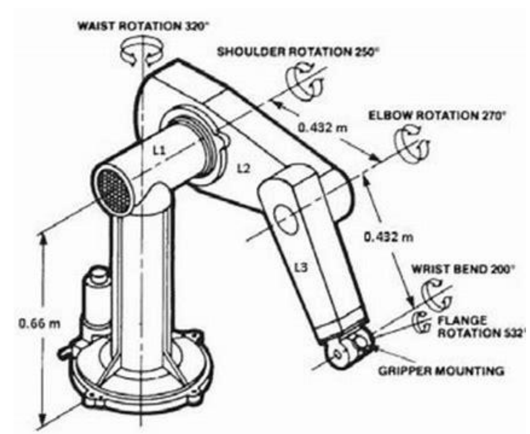
· Arm: (kinematics)

end-effector $y = FK(\theta)$ ⇐ static

· dynamic: ODE / PDE

- **Three pillars of a simulator:**
  1. Constructing the differential equations/models ∈ { ease of use; diversity of env + robot + sensor }
  2. Solving differential equations (accuracy + speed)
     solver / engine (algo)
  3. Visualization of the simulation results





Dynamics equations of Puma 560 Arm

$I_2 = I_{zz2} + m_2*(r_{x2}^2 + r_{y2}^2) + (m_3+m_4+m_5+m_6)*a_2^2\,;$

$I_3 = -I_{xx2} + I_{yy2} + (m_3+m_4+m_5+m_6)*a_2^2$
$\quad m_2*r_{x2}^2 - m_2*r_{y2}^2\,;$

$I_4 = m_2*r_{z2}*(d_2+r_{z2}) + m_3*a_2*r_{z3}$
$\quad +(m_3+m_4+m_5+m_6)*a_2*(d_2+d_3)\,;$

$I_5 = -m_3*a_2*r_{y3} + (m_4+m_5+m_6)*a_2*d_4 + m_4*a_2*r_{z4}\,;$

$I_6 = I_{xx3} + m_3*r_{y3}^2 + m_4*a_3^2 + m_4*(d_4+r_{z4})^2 + I_{yy4}$
$\quad +m_5*a_3^2 + m_5*d_4^2 + I_{zz5} + m_6*a_3^2 + m_6*d_4^2$
$\quad +m_6*r_{z6}^2 + I_{zz6}\,;$

$I_7 = m_3*r_{y3}^2 + I_{zz3} - I_{yy3} + m_4*r_{z4}^2 + 2*m_4*d_4*r_{z4}$
$\quad +(m_4+m_5+m_6)*(d_4^2 - a_3^2) + I_{yy4} - I_{zz4} + I_{zz5}$
$\quad -I_{yy5} + m_6*r_{z6}^2 - I_{zz6} + I_{zz6}\,;$

$I_8 = -m_4*(d_2+d_3)*(d_4+r_{z4}) - (m_5+m_6)*(d_2+d_3)*d_4$
$\quad m_3*r_{y3}*r_{z3} + m_3*(d_2+d_3)*r_{y3}\,;$

$I_9 = m_2*r_{y2}*(d_2+r_{z2})\,;$

$I_{10} = 2*m_4*a_3*r_{z4} + 2*(m_4+m_5+m_6)*a_3*d_4\,;$

$I_{11} = -2*m_2*r_{z2}*r_{y2}\,;$

$I_{12} = (m_4+m_5+m_6)*a_2*a_3\,;$

$I_{13} = (m_4+m_5+m_6)*a_3*(d_2+d_3)\,;$

$I_{14} = I_{zz4} + I_{yy5} + I_{zz6}\,;$

$I_{15} = m_6*d_4*r_{z6}\,;$

$I_{16} = m_6*a_2*r_{z6}\,;$

$I_{17} = I_{zz5} + I_{zz6} + m_6*r_{z6}^2\,;$

$I_{18} = m_6*(d_2+d_3)*r_{z6}\,;$

$I_{19} = I_{yy4} - I_{zz4} + I_{zz5} - I_{yy5} + m_6*r_{z6}^2 + I_{xx6} - I_{zz6}\,;$

$I_{20} = I_{yy5} - I_{zz5} - m_6*r_{z6}^2 + I_{zz6} - I_{xx6}\,;$

$I_{21} = I_{zz4} - I_{yy4} + I_{zz5} - I_{zz5}\,;$

$I_{22} = m_6*a_3*r_{z6}\,;$

$I_{23} = I_{zz6}\,;$

Part II. Gravitional Constants

$g_1 = -g*((m_3+m_4+m_5+m_6)*a_2 + m_2*r_{x2})\,;$

$g_2 = g*(m_3*r_{y3} - (m_4+m_5+m_6)*d_4 - m_4*r_{z4})\,;$

$g_3 = g*m_2*r_{y2}\,;$

$g_4 = -g*(m_4+m_5+m_6)*a_3\,;$

$g_5 = -g*m_6*r_{z6}\,;$

Table A3. Computed Values for the Constants Appearing in the Equations of Forces of Motion.
(Inertial constants have units of kilogram meters-squared)

$I_1 =$	1.43	± 0.05	$I_2 =$	1.75	± 0.07
$I_3 =$	1.38	± 0.05	$I_4 =$	6.90×10^{-1}	$\pm 0.20\times10^{-1}$
$I_5 =$	3.72×10^{-1}	$\pm 0.31\times10^{-1}$	$I_6 =$	3.33×10^{-1}	$\pm 0.16\times10^{-1}$
$I_7 =$	2.98×10^{-1}	$\pm 0.29\times10^{-1}$	$I_8 =$	-1.34×10^{-1}	$\pm 0.14\times10^{-1}$
$I_9 =$	2.38×10^{-2}	$\pm 1.20\times10^{-2}$	$I_{10} =$	-2.13×10^{-2}	$\pm 0.22\times10^{-2}$
$I_{11} =$	-1.42×10^{-2}	$\pm 0.70\times10^{-2}$	$I_{12} =$	-1.10×10^{-2}	$\pm 0.11\times10^{-2}$
$I_{13} =$	-3.79×10^{-3}	$\pm 0.90\times10^{-3}$	$I_{14} =$	1.64×10^{-3}	$\pm 0.07\times10^{-3}$
$I_{15} =$	1.25×10^{-3}	$\pm 0.30\times10^{-3}$	$I_{16} =$	1.24×10^{-3}	$\pm 0.30\times10^{-3}$
$I_{17} =$	6.42×10^{-4}	$\pm 3.00\times10^{-4}$	$I_{18} =$	4.31×10^{-4}	$\pm 1.30\times10^{-4}$
$I_{19} =$	3.00×10^{-4}	$\pm 14.0\times10^{-4}$	$I_{20} =$	-2.02×10^{-4}	$\pm 8.00\times10^{-4}$
$I_{21} =$	-1.00×10^{-4}	$\pm 6.00\times10^{-4}$	$I_{22} =$	-5.80×10^{-5}	$\pm 1.50\times10^{-5}$
$I_{23} =$	4.00×10^{-5}	$\pm 2.00\times10^{-5}$			
$I_{m1} =$	1.14	± 0.27	$I_{m2} =$	4.71	± 0.54
$I_{m3} =$	8.27×10^{-1}	$\pm 0.93\times10^{-1}$	$I_{m4} =$	2.00×10^{-1}	$\pm 0.16\times10^{-1}$
$I_{m5} =$	1.79×10^{-1}	$\pm 0.14\times10^{-1}$	$I_{m6} =$	1.93×10^{-1}	$\pm 0.16\times10^{-1}$

(Gravitational constants have units of newton meters)

$g_1 =$	-37.2	± 00.5	$g_2 =$	-8.44	± 0.20
$g_3 =$	1.02	± 0.50	$g_4 =$	2.49×10^{-1}	$\pm 0.25\times10^{-1}$
$g_5 =$	-2.82×10^{-2}	$\pm 0.56\times10^{-2}$			

$b_{224} = 2*\{-I_{16}*C3*S4*S5 + I_{20}*SC4*SS5$
$\quad +I_{21}*SC4 - I_{22}*S4*S5\}\,;$
$\quad \approx -2.48\times10^{-3}*C3*S4*S5\,.$

$b_{225} = 2*\{-I_{15}*S5 + I_{16}*(C3*C4*C5 - S3*S5)$
$\quad +I_{20}*SS4*SC5 + I_{22}*C4*C5\}\,;$
$\quad \approx -2.50\times10^{-3}*S5 + 2.48\times10^{-3}*(C3*C4*C5 - S3*S5)\,.$

$b_{226} = 0\,. \qquad b_{234} = b_{224}\,.$

$b_{235} = b_{225}\,. \qquad b_{236} = 0\,.$

$b_{245} = 2*\{-I_{15}*S4*C5 - I_{16}*S3*S4*C5\}$
$\quad -I_{17}*S4 + I_{20}*S4*(1-2*SS5)\,; \qquad \approx 0\,.$

$b_{246} = I_{23}*C4*S5\,; \qquad \approx 0\,.$

$b_{256} = I_{23}*S4*C5\,; \qquad \approx 0\,.$

$b_{312} = 0\,. \qquad b_{313} = 0\,.$

$b_{314} = 2*\{-I_{15}*C23*C4*S5 + I_{22}*S23*C4*S5$
$\quad +I_{20}*(S23*(CC5*CC4 - 0.5) + C23*C4*SC5)\}$
$\quad +I_{14}*S23 + I_{19}*S23*(1-(2*SS4))\,;$
$\quad \approx -2.50\times10^{-3}*C23*C4*S5 + 1.64\times10^{-3}*S23$
$\quad + 0.30\times10^{-3}*S23*(1-2*SS4)\,.$

$b_{315} = 2*(-I_{15}*C23*S4*C5 + I_{22}*S23*S4*C5)$
$\quad -I_{17}*C23*S4$
$\quad +I_{20}*S4*(C23*(1-2*SS5) - 2*S23*C4*SC5)\,;$
$\quad \approx -2.50\times10^{-3}*C23*S4*C5 - 6.42\times10^{-4}*C23*S4\,.$

$b_{316} = -b_{136}\,. \qquad b_{323} = 0\,.$

$b_{324} = 2*\{I_{20}*SC4*SS5 + I_{21}*SC4 - I_{22}*S4*S5\}\,;$
$\quad \approx 0\,.$

$b_{325} = 2*\{-I_{15}*S5 + I_{20}*SS4*SC5 + I_{22}*C4*C5\}\,;$
$\quad \approx -2.50\times10^{-3}*S5\,.$

$b_{326} = 0\,. \qquad b_{334} = b_{324}\,.$

$b_{335} = b_{325}\,. \qquad b_{336} = 0\,.$

$b_{345} = -I_{15}*2*S4*C5 - I_{17}*S4 + I_{20}*S4*(1-2*SS5)\,;$
$\quad \approx -2.50\times10^{-3}*S4*C5\,.$

$b_{346} = b_{246}\,. \qquad b_{356} = b_{256}\,. \qquad b_{412} = -b_{214}\,.$

$b_{413} = -b_{314}\,. \qquad b_{414} = 0\,.$

$b_{415} = -I_{20}*(S23*C4*(1-2*SS5) + 2*C23*SC5)$
$\quad -I_{17}*S23*C4\,;$
$\quad \approx -6.42\times10^{-4}*S23*C4\,.$

$b_{416} = -b_{146}\,. \qquad b_{423} = -b_{324}\,. \qquad b_{424} = 0\,.$

$b_{425} = I_{17}*S4 + I_{20}*S4*(1-2*SS5)\,;$
$\quad \approx 6.42\times10^{-4}*S4\,.$

$b_{426} = -b_{246}\,. \qquad b_{434} = 0\,.$

$b_{435} = b_{425}\,. \qquad b_{436} = -b_{346}\,.$

$b_{445} = -I_{20}*2*SC5\,; \qquad \approx 0\,.$

$b_{446} = 0\,;$

$b_{456} = -I_{23}*S5\,; \qquad \approx 0\,.$

$b_{512} = -b_{215}\,. \qquad b_{513} = -b_{315}\,. \qquad b_{514} = -b_{415}\,.$

$b_{515} = 0\,. \qquad b_{516} = -b_{156}\,. \qquad b_{523} = -b_{325}\,.$

$b_{524} = -b_{425}\,. \qquad b_{525} = 0\,. \qquad b_{526} = -b_{256}\,.$

$b_{534} = b_{524}\,. \qquad b_{535} = 0\,. \qquad b_{536} = -b_{356}\,.$

$b_{545} = 0\,. \qquad b_{546} = -b_{456}\,. \qquad b_{556} = 0\,.$

$b_{612} = b_{126}\,. \qquad b_{613} = b_{136}\,. \qquad b_{614} = b_{146}\,.$

$b_{615} = b_{156}\,. \qquad b_{616} = 0\,. \qquad b_{623} = 0\,.$

$b_{624} = b_{246}\,. \qquad b_{625} = b_{256}\,. \qquad b_{626} = 0\,.$

$b_{634} = b_{624}\,. \qquad b_{635} = b_{625}\,. \qquad b_{636} = 0\,.$

$b_{645} = b_{456}\,. \qquad b_{646} = 0\,. \qquad b_{656} = 0\,.$

Table A6. The expressions for the terms of the centrifugal matrix.
(The Abbreviated Expressions have units of kg-m^2.)

$c_{11} = 0\,.$

$c_{12} = +I4*C2 - I8*S23 - I9*S2 + I13*C23$
$\quad +I_{15}*S23*S4*S5 + I_{16}*C2*S4*S5$
$\quad +I_{18}*(C23*C4*S5 + S23*C5) + I_{19}*C23*SC4$
$\quad +I_{20}*S4*(C23*C4*cC5 - S23*SC5)$
$\quad +I_{22}*C23*S4*S5\,;$
$\approx 6.90\times10^{-1}*C2 + 1.34\times10^{-1}*S23 - 2.38\times10^{-2}*S2\,.$

$c_{13} = 0.5*b_{123}\,.$

$c_{14} = -I_{15}*S23*S4*S5 - I_{16}*C2*S4*S5$
$\quad +I_{18}*C23*C4*S5 + I_{20}*S23*S4*SC5$
$\quad -I_{22}*C23*S4*S5\,; \qquad \approx 0\,.$

$c_{15} = -I_{15}*S23*S4*S5 - I_{16}*C2*S4*S5\,;$
$\quad +I_{18}*(S23*C5 + C23*C4*S5) - I_{22}*C23*S4*S5$
$\approx 0\,.$

$c_{16} = 0\,. \qquad c_{21} = -0.5*b_{112}\,.$

$c_{22} = 0\,. \qquad c_{23} = 0.5*b_{223}\,.$

$c_{24} = -I_{15}*C4*S5 - I_{16}*S3*C4*S5 + I_{20}*C4*SC5\,;$
$\approx 0\,.$

$c_{25} = -I_{15}*C4*S5 + I_{16}*(C3*C5 - S3*C4*S5)$
$\quad +I_{22}*C5\,; \qquad \approx 0\,.$

$c_{26} = 0\,. \qquad c_{31} = -0.5*b_{113}\,.$

$c_{32} = -c_{23}\,. \qquad c_{33} = 0\,.$

$c_{34} = -I_{15}*C4*S5 + I_{20}*C4*SC5\,;$
$\approx -1.25\times10^{-3}*C4*S5\,.$

$c_{35} = -I_{15}*C4*S5 + I_{22}*C5\,; \qquad \approx c_{34}\,.$

$c_{36} = 0\,. \qquad c_{41} = -0.5*b_{114}\,. \qquad c_{42} = -0.5*b_{224}\,.$

$c_{43} = 0.5*b_{423}\,. \qquad c_{44} = 0\,. \qquad c_{45} = 0\,.$

$c_{46} = 0\,. \qquad c_{51} = -0.5*b_{115}\,. \qquad c_{52} = -0.5*b_{225}\,.$

$c_{53} = 0.5*b_{523}\,. \qquad c_{54} = -0.5*b_{445}\,. \qquad c_{55} = 0\,.$

$c_{56} = 0\,. \qquad c_{61} = 0\,. \qquad c_{62} = 0\,.$

$c_{63} = 0\,. \qquad c_{64} = 0\,. \qquad c_{65} = 0\,.$

$c_{66} = 0\,.$

Table A7. Gravity Terms.
(The Abbreviated Expressions have units of newton-meters.)

$g_1 = 0\,.$

$g_2 = g1*C2 + g2*S23 + g3*S2 + g4*C23\,;$
$\quad +g5*(S23*C5 + C23*C4*S5)$
$\approx -37.2*C2 - 8.4*S23 + 1.02*S2\,.$

$g_3 = g2*S23 + g4*C23 + g5*(S23*C5 + C23*C4*S5)\,;$
$\approx -8.4*S23 + 0.25*C23\,.$

$g_4 = -g5*S23*S4*S5\,;$
$\approx 2.8\times10^{-2}*S23*S4*S5\,.$

$g_5 = g5*(C23*S5 + S23*C4*C5)\,;$
$\approx -2.8\times10^{-2}*(C23*S5 + S23*C4*C5)\,.$

$g_6 = 0\,.$

Table A4. The expressions giving the elements of the kinetic energy matrix.
(The Abbreviated Expressions have units of kg-m^2.)

$a_{11} = I_{m1} + I_1 + I_3*CC2 + I_7*SS23 + I_{10}*SC23 + I_{11}*SC2$
$\quad +I_{20}*(SS5*(SS23*(1+CC4) - 1) - 2*SC23*C4*SC5)$
$\quad +I_{21}*SS23*CC4 + 2*\{I_5*C2*S23 + I_{12}*C2*C23$
$\quad +I_{15}*(SS23*C5 + SC23*C4*S5)$
$\quad +I_{16}*C2*(S23*C5 + C23*C4*S5)$
$\quad +I_{18}*S4*S5 + I_{22}*(SC23*C5 + CC23*C4*S5)\}\,;$
$\approx 2.57 + 1.38*CC2 + 0.30*SS23 + 7.44\times10^{-1}*C2*S23\,.$

$a_{12} = I_4*S2 + I_8*C23 + I_9*C2 + I_{13}*S23 - I_{15}*C23*S4*S5$
$\quad +I_{16}*S2*S4*S5 + I_{18}*(S23*C4*S5 - C23*C5)$
$\quad +I_{19}*S23*SC4 + I_{20}*S4*(S23*C4*CC5 + C23*SC5)$
$\quad +I_{22}*S23*S4*S5\,;$
$\approx 6.90\times10^{-1}*S2 - 1.34\times10^{-1}*C23 + 2.38\times10^{-2}*C2\,.$

$a_{13} = I_8*C23 + I_{13}*S23 - I_{15}*C23*S4*S5 + I_{19}*S23*SC4$
$\quad +I_{18}*(S23*C4*S5 - C23*C5) + I_{22}*S23*S4*S5$
$\quad +I_{20}*S4*(S23*C4*CC5 + C23*SC5)\,;$
$\approx -1.34\times10^{-1}*C23 + -3.97\times10^{-3}*S23\,.$

$a_{14} = I_{14}*C23 + I_{15}*S23*C4*S5 + I_{16}*C2*C4*S5$
$\quad +I_{18}*C23*S4*S5 - I_{20}*(S23*C4*SC5 + C23*SS5)$
$\quad +I_{22}*C23*C4*S5\,; \qquad \approx 0\,.$

$a_{15} = I_{15}*S23*S4*C5 + I_{16}*C2*S4*C5 + I_{17}*S23*S4$
$\quad +I_{18}*(S23*S5 - C23*C4*C5) + I_{22}*C23*S4*C5\,;$
$\approx 0\,.$

$a_{16} = I_{23}*(C23*C5 - S23*C4*S5)\,; \qquad \approx 0\,.$

$a_{22} = I_{m2} + I_2 + I_6 + I_{20}*SS4*SS5 + I_{21}*SS4$
$\quad +2*\{I_5*S3 + I_{12}*C3 + I_{15}*C5$
$\quad +I_{16}*(S3*C5 + C3*C4*S5) + I_{22}*C4*S5\}\,;$
$\approx 6.79 + 7.44\times10^{-1}*S3\,.$

$a_{23} = I_5*S3 + I_6 + I_{12}*C3 + I_{16}*(S3*C5 + C3*C4*S5)$
$\quad +I_{20}*SS4*SS5 + I_{21}*SS4 + 2*\{I_{15}*C5 + I_{22}*C4*S5\}\,;$
$\approx .333 + 3.72\times10^{-1}*S3 - 1.10\times10^{-2}*C3\,.$

$a_{24} = -I_{15}*S4*S5 - I_{16}*S3*S4*S5 + I_{20}*S4*SC5\,;$
$\approx 0\,.$

$a_{25} = I_{15}*C4*C5 + I_{16}*(C3*S5 + S3*C4*C5)$
$\quad +I_{17}*C4 + I_{22}*S5\,; \qquad \approx 0\,.$

$a_{26} = I_{23}*S4*S5\,; \qquad \approx 0\,.$

$a_{33} = I_{m3} + I_6 + I_{20}*SS4*SS5 + I_{21}*SS4$
$\quad +2*\{I_{15}*C5 + I_{22}*C4*S5\}\,; \qquad \approx 1.16\,.$

$a_{34} = -I_{15}*S4*S5 + I_{20}*S4*SC5\,;$
$\approx -1.25\times10^{-3}*S4*S5\,.$

$a_{35} = I_{15}*C4*C5 + I_{17}*C4 + I_{22}*S5\,;$
$\approx 1.25\times10^{-3}*C4*C5\,.$

$a_{36} = I_{23}*S4*S5\,; \qquad \approx 0\,.$

$a_{44} = I_{m4} + I_{14} - I_{20}*SS5\,; \qquad \approx 0.20\,.$

$a_{45} = 0.$

$a_{46} = I_{23}*C5\,; \qquad \approx 0\,.$

$a_{55} = I_{m5} + I_{17}\,; \qquad \approx 0.18\,.$

$a_{56} = 0.$

$a_{66} = I_{m6} + I_{23}\,; \qquad \approx 0.19\,.$

Table A5. The expressions giving the elements of the Coriolis matrix.
(The Abbreviated Expressions have units of kg-m^2.)

$b_{112} = 2*\{-I_3*SC2 + I_5*C223 + I_7*SC23 - I_{12}*S223$
$\quad +I_{15}*(2*SC23*C5 + (1-2*SS23)*C4*S5)$
$\quad +I_{16}*(C223*C5 - S223*C4*S5) + I_{21}*SC23*CC4$
$\quad +I_{20}*((1+CC4)*SC23*SS5 - (1-2*SS23)*C4*SC5)$
$\quad +I_{22}*((1-2*SS23)*C5 - 2*SC23*C4*S5)\}$
$\quad +I_{10}*(1-2*SS23) + I_{11}*(1-2*SS2)\,;$
$\approx -2.76*SC2 + 7.44\times10^{-1}*C223 + 0.60*SC23$
$\quad - 2.13\times10^{-2}*(1-2*SS23)\,.$

$b_{113} = 2*\{I_5*C2*C23 + I_7*SC23 - I_{12}*C2*S23$
$\quad +I_{15}*(2*SC23*C5 + (1-2*SS23)*C4*S5)$
$\quad +I_{16}*C2*(C23*C5 - S23*C4*S5) + I_{21}*SC23*CC4$
$\quad +I_{20}*((1+CC4)*SC23*SS5 - (1-2*SS23)*C4*SC5)$
$\quad +I_{22}*((1-2*SS23)*C5 - 2*SC23*C4*S5)\}$
$\quad +I_{10}*(1-2*SS23)\,;$
$\approx 7.44\times10^{-1}*C2*C23 + 0.60*SC23$
$\quad + 2.20\times10^{-2}*C2*S23 - 2.13\times10^{-2}*(1-2*SS23)\,.$

$b_{114} = 2*\{-I_{15}*SC23*S4*S5 - I_{16}*C2*C23*S4*S5$
$\quad +I_{18}*C4*S5 - I_{20}*(SS23*SS5*SC4 - SC23*S4*SC5)$
$\quad -I_{22}*CC23*S4*S5 - I_{21}*SS23*SC4\}\,;$
$\approx -2.50\times10^{-3}*SC23*S4*S5 + 8.60\times10^{-4}*C4*S5$
$\quad - 2.48\times10^{-3}*C2*C23*S4*S5\,.$

$b_{115} = 2*\{I_{20}*(SC5*(CC4*(1-CC23) - CC23)$
$\quad -SC23*C4*(1-2*SS5)) - I_{15}*(SS23*S5 - SC23*C4*C5)$
$\quad -I_{16}*C2*(S23*S5 - C23*C4*C5) + I_{18}*S4*C5$
$\quad +I_{22}*(CC23*C4*C5 - SC23*S5)\}\,;$
$\approx -2.50\times10^{-3}*(SS23*S5 - SC23*C4*C5)$
$\quad - 2.48\times10^{-3}*C2*(S23*S5 - C23*C4*C5)$
$\quad + 8.60\times10^{-4}*S4*C5.$

$b_{116} = 0\,.$

$b_{123} = 2*\{-I_8*S23 + I_{13}*C23 + I_{15}*S23*S4*S5$
$\quad +I_{18}*(C23*C4*S5 + S23*C5) + I_{19}*C23*SC4$
$\quad +I_{20}*S4*(C23*C4*CC5 - S23*SC5)$
$\quad +I_{22}*C23*S4*S5\}\,;$
$\approx 2.67\times10^{-1}*S23 - 7.58\times10^{-3}*C23\,.$

$b_{124} = -I_{18}*2*S23*S4*S5 + I_{19}*S23*(1-(2*SS4))$
$\quad +I_{20}*S23*(1-2*SS4*CC5) - I_{14}*S23\,; \qquad \approx 0\,.$

$b_{125} = I_{17}*C23*S4 + I_{18}*2*(S23*C4*C5 + C23*S5)$
$\quad +I_{20}*S4*(C23*(1-2*SS5) - S23*C4*2*SC5)\,;$
$\approx 0\,.$

$b_{126} = -I_{23}*(S23*C5 + C23*C4*S5)\,; \qquad \approx 0\,.$

$b_{134} = b_{124}\,. \qquad b_{135} = b_{125}\,. \qquad b_{136} = b_{126}\,.$

$b_{145} = 2*\{I_{15}*S23*C4*C5 + I_{16}*C2*C4*C5$
$\quad +I_{18}*C23*S4*C5 + I_{22}*C23*C4*C5\} + I_{17}*S23*C4$
$\quad -I_{20}*(S23*C4*(1-2*SS5) + 2*C23*SC5)\,;$
$\approx 0\,.$

$b_{146} = I_{23}*S23*S4*S5\,; \qquad \approx 0\,.$

$b_{156} = -I_{23}*(C23*S5 + S23*C4*C5)\,; \qquad \approx 0\,.$

$b_{212} = 0\,. \qquad b_{213} = 0\,.$

$b_{214} = I_{14}*S23 + I_{19}*S23*(1-(2*SS4))$
$\quad +2*\{-I_{15}*C23*C4*S5 + I_{16}*S2*C4*S5$
$\quad +I_{20}*(S23*(CC5*CC4 - 0.5) + C23*C4*SC5)$
$\quad +I_{22}*S23*C4*S5\}\,;$
$\approx 1.64\times10^{-3}*S23 - 2.50\times10^{-3}*C23*C4*S5 +$
$\quad 2.48\times10^{-3}*S2*C4*S5 + 0.30\times10^{-3}*S23*(1-(2*SS4))\,.$

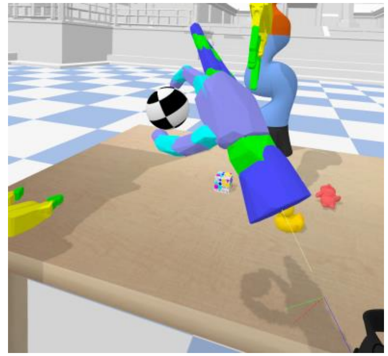
$b_{215} = 2*\{-I_{15}*C23*S4*C5 + I_{22}*S23*S4*C5$
$\quad +I_{16}*S2*S4*C5\} - I_{17}*C23*S4$
$\quad +I_{20}*(C23*S4*(1-2*SS5) - 2*S23*SC4*SC5)\,;$
$\approx -2.50\times10^{-3}*C23*S4*C5 + 2.48\times10^{-3}*S2*S4*C5$
$\quad - 6.42\times10^{-4}*C23*S4\,.$

$b_{216} = -b_{126}\,.$

$b_{223} = 2*\{-I_{12}*S3 + I_5*C3 + I_{16}*(C3*C5 - S3*C4*S5)\}\,;$
$\approx 2.20\times10^{-2}*S3 + 7.44\times10^{-1}*C3\,.$

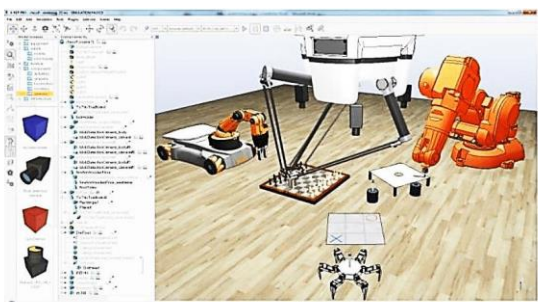
Popular simulators in robotics

Mujoco (Roboti LLC)



PyBullet (open source)

Isaac Sim (NVIDIA)

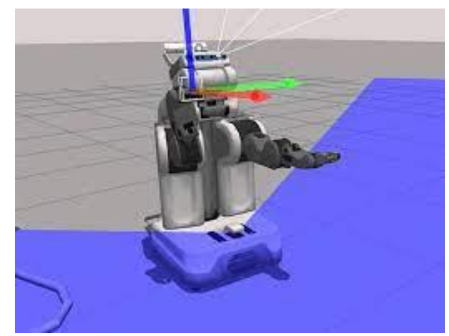


V-REP (CoppeliaSim)

DRAKE

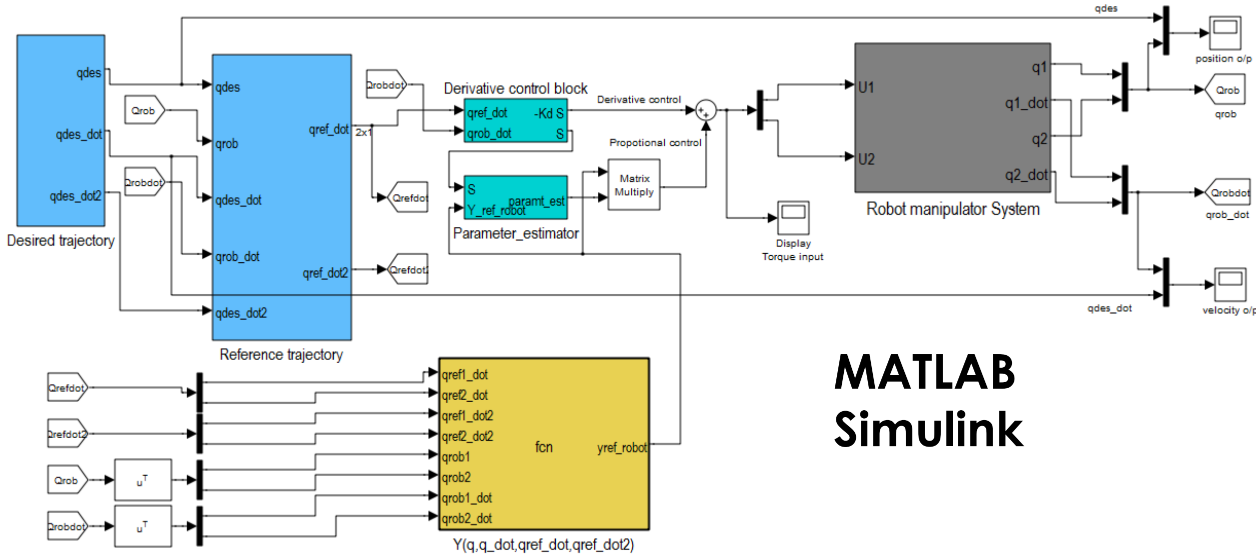
A PLANNING, CONTROL, AND ANALYSIS TOOLBOX FOR NONLINEAR DYNAMICAL SYSTEMS

Drake(Open source)



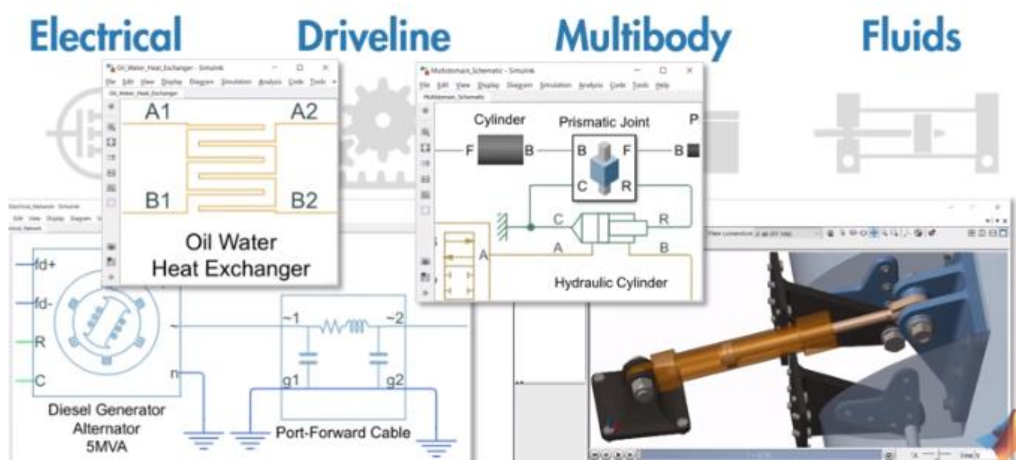
Gazebo

- Popular simulators for control systems



MATLAB Simulink

MATLAB Simscape



Outline

- Short introduction to Simulation
- **Introduction to Mujoco**
- Python Example

▪ Mujoco

- **High-Performance Physics Engine**: MuJoCo offers highly accurate simulations of complex physical interactions, ideal for robotics research
- **Fast Real-Time Simulations**: Its optimization allows for real-time performance, making it suitable for reinforcement learning applications
- **Advanced Contact Dynamics**: MuJoCo handles contact and friction with soft constraints, providing realistic interactions in dynamic environments.
- **Drawbacks:**
    - Lacks of detailed sensor models
    - Struggle with large or highly diverse environments

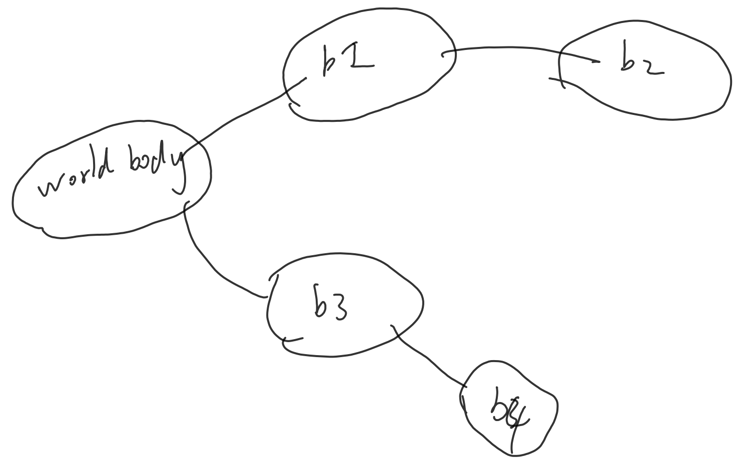
▪ How to define a robot control system?

```
xml4="""<mujoco model="3R_robot">
    <compiler angle="degree"/>
    <asset>
        <texture name="grid" type="2d" builtin="checker" rgb1=".1 .2 .3"
        rgb2=".2 .3 .4" width="300" height="300" mark="none"/>
        <material name="grid" texture="grid" texrepeat="6 6" texuniform="true" reflectance=".2"/>
    </asset>
    <default>
        <joint type="hinge" axis="0 0 1" limited="true"/>
        <geom type="cylinder" size=".025 .1" />
    </default>

    <worldbody>
        <light diffuse=".5 .5 .5" pos="0 0 3" dir="0 0 -1"/>
        <geom type="plane" size="1 1 0.1" material="grid"/>

        <body name="BaseLink" pos="0 0 0.1">
            <geom type="cylinder" pos="0 0 0" size=".025 .1" />
            <body name="link1" pos="0 0.1 0.125" euler="-90 0 0">
                <joint name="joint1" pos="0 0 -0.1" range="-90 90" axis ="0 1 0"/>
                <geom pos="0 0 0" rgba=".6 .2 .2 1"/>
                <site name="torque_site" pos="0 0.2 0"/>
                <body name="link2" pos="0 0 0.2">
                    <joint name="joint2" pos="0 0 -0.1" range="-90 90" axis="0 1 0"/>
                    <geom rgba=".2 .6 1 1"/>
                    <site name="end_effector" pos="0 0 0.1" size="0.01"/>
                </body>
            </body>
        </body>
    </worldbody>
</mujoco>"""
```

```
import mujoco
m = mujoco.MjModel.from_xml_string(xml4)
m = mujoco.MjModel.from_xml_path('***.xml')
d = mujoco.MjData(m)
```

How to define a robot control system?

- We focus on **rigid body** system: Multiple rigid bodies interconnected through joints.



How to define **a rigid body**?

1. Where is it? (pos / orientation

<body> body frame

2. How does it look?

<geom>

3. How does it connect to others?

<joint>

<option>

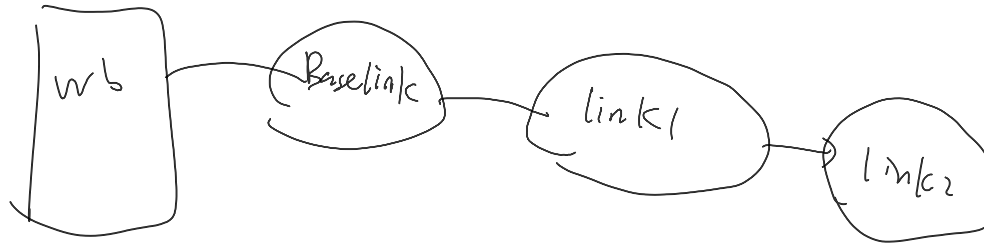
4. Its physical properties?

<Inertia> ${}^{A}I$ ${}^{B}I$

Inertia frame

▪ How to define a rigid body?

1. Where is it?



- body element:
  - <name>: optional
  - <pos>:
  - <euler>, or <quat> or <axisangle>: specify frame orientation relative to parent frame, optional (default orientation matrix is identity)

```
<worldbody>
    <body name="BaseLink" pos="0 0 0.1">
        <body name="link1" pos="0 0.1 0.125" euler="-90 0 0">
            <body name="link2" pos="0 0 0.2">
            </body>
        </body>
    </body>
</worldbody>
```

▪ How to define a rigid body?

2. How does it look?

geom: sub-element of body

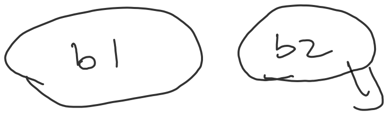
→ body frame

- <name> (optional), position <pos>, orientation
- <type>: sphere (default), plane, capsule, ellipsoid, cylinder, box, mesh, sdf

Type	Number	Description
plane	3	X half-size; Y half-size; spacing between square grid lines for rendering. If either the X or Y half-size is 0, the plane is rendered as infinite in the dimension(s) with 0 size.
hfield	0	The geom sizes are ignored and the height field sizes are used instead.
sphere	1	Radius of the sphere.
capsule	1 or 2	Radius of the capsule; half-length of the cylinder part when not using the fromto specification.
ellipsoid	3	X radius; Y radius; Z radius.
cylinder	1 or 2	Radius of the cylinder; half-length of the cylinder when not using the fromto specification.
box	3	X half-size; Y half-size; Z half-size.
mesh	0	The geom sizes are ignored and the mesh sizes are used instead.

- <fromto>:
- <material>:
- <rgba>:
- <mass>: optional
- <density>: default “1000”: density of water in SI unit

▪ How to define a rigid body?



3. How does it connect to others?

- joint: sub-element of body

(if no <joint> of b2, then b2 is welded to b1)

world body no joint

- <name>
- type: "free": 6 dof
  - "slide": linear
  - "hinge": rotation.
- pos
- axis: rotation axis for "hinge", direction of translation for "slide".

▪ How to define a rigid body?

4. Its physical properties?

Type 1 (default): infer from geom attached to the body

Type 2: inertia sub-element

- <pos>: position of inertial frame.
- <orientation>: of the inertial frame
- <mass>: positive number required
- <diaginertia> (real(3)): diagonal entries of the inertial matrix;
- <fullinertia>: real(6): Full inertia matrix M: M(1,1), M(2,2), M(3,3), M(1,2), M(1,3), M(2,3).

```xml
<mujoco>
  <worldbody>
    <body name="arm" pos="0 0 0">
      <!-- Define the mass and inertia of this body -->
      <inertial pos="0 0 0" mass="1.0" diaginertia="0.01 0.01 0.01"/>

      <!-- Additional parts of the body, like joints or geometric shapes
      <geom type="capsule" size="0.05 0.2" rgba="0.8 0.3 0.3 1" />
      <joint type="hinge" axis="0 1 0" />
    </body>
  </worldbody>
</mujoco>
```

Assets

Assets are not in themselves model elements. Model elements can reference them. One asset can be referenced by multiple model elements.

- **asset/mesh:** MuJoCo works with triangulated meshes. They can be loaded from binary STL files, OBJ files or MSH files.

- **asset/material:** It can be referenced from skins, geoms, sites and tendons to set their appearance. Materials are useful for adjusting appearance properties beyond color.

```
<mujoco>
  <asset>
    <texture name="grid" type="2d" builtin="checker" rgb1=".1 .2 .3"
     rgb2=".2 .3 .4" width="300" height="300" mark="none"/>
    <material name="grid" texture="grid" texrepeat="6 6"
     texuniform="true" reflectance=".2"/>
     <material name="wall" rgba='.5 .5 .5 1'/>
  </asset>
  <default>
    <geom type="box" size=".05 .05 .05" />
    <joint type="free"/>
  </default>

  <worldbody>
    <light name="light" pos="-.2 0 1"/>
    <geom name="ground" type="plane" size="10 10 10" material="grid"
     zaxis="-.3 0 1" friction=".5"/>
    <camera name="y" pos="-.1 -.6 .3" xyaxes="1 0 0 0 1 2"/>
    <body pos="0 1 .3">
      <joint/>
      <geom friction="0.3"/>
    </body>
    <body pos="0 0 .3">
      <joint/>
      <geom friction="1"/>
    </body>
  </worldbody>
```

<default>

<geom type="box" >

Outline

- Short introduction to Simulation
- Introduction to Mujoco
- **Python Example**

- Summary