

SDM5008 Advanced Control for Robotics

Lecture 7: Rigid Body Dynamics

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Outline

- ~~Spatial Acceleration~~ ✓
- Spatial Force (Wrench)
- Spatial Momentum
- Newton Euler Equation in Spatial Algebra ✓

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Spatial Force (Wrench)

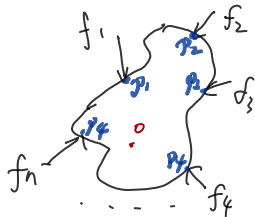
- Consider a rigid body with many forces on it and fix an arbitrary point O in space

- The net effect of these forces can be expressed as
 - A force f , acting along a line passing through O

$$f = \sum_i f_i$$

- A moment n_O about point O

$$n_O = \sum_{i=1}^n \vec{op}_i \times f_i$$



- Spatial Force (Wrench):** is given by the 6D vector

$$F = \begin{bmatrix} n_O \\ f \end{bmatrix} \leftarrow \text{wrench}$$

What if we change reference point from " O " to " q ": by definition.

$$n_q = \sum_i \vec{qp}_i \times f_i = n_O + \sum_i (\vec{qp}_i - \vec{op}_i) \times f_i = n_O + \sum_i \vec{qo} \times f_i$$

Spatial Force in Plücker Coordinate Systems

- Given a frame $\{A\}$, the Plücker coordinate of a spatial force \mathcal{F} is given by

$${}^A\mathcal{F} = \begin{bmatrix} {}^A n_{o_A} \\ {}^A f \end{bmatrix}$$

- Coordinate transform: ${}^A\mathcal{F} = {}^A X_B^* {}^B\mathcal{F}$ where ${}^A X_B^* = {}^B X_A^T$

Wrench-Twist Pair and Power

- Recall that for a point mass with linear velocity v and linear force f . Then we know that the power (instantaneous work done by f) is given by $f \cdot v = f^T v$
- This relation can be generalized to spatial force (i.e. wrench) and spatial velocity (i.e. twist)
- Suppose a rigid body has a twist ${}^A\mathcal{V} = ({}^A\omega, {}^A v_{O_A})$ and a wrench ${}^A\mathcal{F} = ({}^A n_{O_A}, {}^A f)$ acts on the body. Then the power is simply

$$P = ({}^A\mathcal{V})^T {}^A\mathcal{F}$$

Joint Torque

- Consider a link attached to a 1-dof joint (e.g. revolute or prismatic). Let $\hat{\mathcal{S}}$ be the screw axis of the joint. The velocity of the link induced by joint motion is given by: $\mathcal{V} = \hat{\mathcal{S}}\dot{\theta}$

- \mathcal{F} be the wrench provided by the joint. Then the power produced by the joint is

$$P = \mathcal{V}^T \mathcal{F} = (\hat{\mathcal{S}}^T \mathcal{F})\dot{\theta} \triangleq \tau\dot{\theta}$$

- $\tau = \hat{\mathcal{S}}^T \mathcal{F} = \mathcal{F}^T \hat{\mathcal{S}}$ is the projection of the wrench onto the screw axis, i.e. the effective part of the wrench.
- Often times, τ is referred to as joint "torque" or generalized force

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- Spatial Force (Wrench)
- Spatial Momentum

Rotational Inertia (1/2)

- Recall momentum for point mass:

Rotational Inertia (2/2)

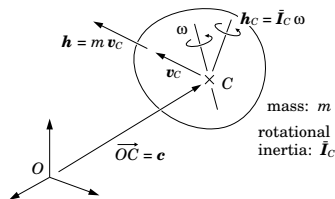
- Rotational Inertia: $\bar{I} = \int_V \rho(r)[r][r]^T dr$
 - $\rho(\cdot)$ is the density function of the body

 - \bar{I} depends on coordinate system

 - It is a constant matrix if the origin coincides with CoM

Spatial Momentum

- Consider a rigid body with spatial velocity $\mathcal{V}_C = (\omega, v_C)$ expressed at the center of mass C
 - Linear momentum:
 - Angular momentum about CoM:
 - Angular momentum about a point O :
- Spatial Momentum:



Change Reference Frame for Momentum

- Spatial momentum transforms in the same way as spatial forces:

$${}^A h = {}^A X_C^* {}^C h$$

Spatial Inertia

- Inertia of a rigid body defines linear relationship between velocity and momentum.
- Spatial inertia \mathcal{I} is the one such that

$$h = \mathcal{I}\mathcal{V}$$

- Let $\{C\}$ be a frame whose origin coincide with CoM. Then

$${}^c\mathcal{I} = \begin{bmatrix} {}^c\bar{I}_c & 0 \\ 0 & mI_3 \end{bmatrix}$$

Spatial Inertia

- Spatial inertia wrt another frame $\{A\}$:

$${}^A\mathcal{I} = {}^AX_C^* {}^C\mathcal{I} {}^CX_A$$

- Special case: ${}^AR_C = I_3$

More Discussions

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