

SDM5008 Advanced Control for Robotics

Lecture 4: Instantaneous Velocity of Moving Frames

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Outline

$$\underbrace{[W] = \dot{R}R^{-1}}_{R^{-1}\dot{R}} \quad \underbrace{[V] = \dot{T}T^{-1}}$$

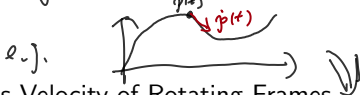
- Instantaneous Velocity of Rotating Frames

- Instantaneous Velocity of Moving Frames

Outline - $T_A(t) \in \mathbb{R}^{4 \times 4}$: 4×4 matrix $\in SE(3)$



$\log(T_A(t)) \rightarrow \mathfrak{se}(3)$: is $\mathfrak{se}(3)$ the velocity of $T_A(t)$?
 \rightarrow No



• Instantaneous Velocity of Rotating Frames

- $p(t)$: "position" vector $\leftrightarrow T_A(t)$
 $\downarrow \updownarrow$
 $\mathfrak{se}(3)$

• Instantaneous Velocity of Moving Frames

- $\dot{p}(t)$: velocity vector $\leftrightarrow ?$

$\dot{T}_A(t)$ is velocity of $T(t)$?
No
 - How about $\log(\dot{T}_A(t))$? \times
 $\notin \mathfrak{se}(3)$

Objectives

Instantaneous Velocity of Rotating Frame (1/2)

- $\{A\}$ frame is rotating with orientation $R_A(t)$ and velocity $\underline{\omega}_A(t)$ at time t
(Note: everything is wrt $\{O\}$ -frame)

$$\hat{\omega}\theta \neq \int \omega_A(t)$$

- Let $\hat{\omega}\theta = \log(\overset{\in SO(3)}{R_A(t)})$ be its exp. coordinate. ("position" - vel)
 - Note: $\hat{\omega}\theta$ means $R_A(t)$ can be obtained from the reference frame (say $\{O\}$ -frame) by rotating about $\hat{\omega}$ by θ degree.

- $\hat{\omega}\theta$ only describes the current orientation of $\{A\}$ relative to $\{O\}$, it does not contain info about how the frame is rotating at time t .

Instantaneous Velocity of Rotating Frame (2/2)

- What is the relation between $\omega_A(t)$ and $R_A(t)$?

$$\frac{d}{dt}R_A(t) = [\omega_A(t)]R_A(t) \Rightarrow [\omega_A(t)] = \dot{R}_A(t)R_A^{-1}(t)$$

Outline

- Instantaneous Velocity of Rotating Frames
- Instantaneous Velocity of Moving Frames

Instantaneous Velocity of Moving Frame (1/2)

- $\{A\}$ moving frame with configuration $T_A(t)$ at time t undergoes a rigid body motion with velocity $\mathcal{V}_A(t) = (\omega, v)$ (Note: everything is wrt $\{O\}$ -frame)

- The exponential coordinate $\hat{S}\theta = \log(T_A(t))$ only indicates the current configuration of $\{A\}$, and does not tell us about how the frame is moving at time t .

Instantaneous Velocity of Moving Frame (2/2)

- What is the relation between $\mathcal{V}_A(t)$ and $T_A(t)$?

$$\frac{d}{dt}T_A(t) = [\mathcal{V}_A(t)]T_A(t) \Rightarrow [\mathcal{V}_A(t)] = \dot{T}_A(t)T_A^{-1}(t)$$

More Space