Fall 2004 SDM5008 Advanced Control for Robotics

Lecture Note 8: Mujoco Tutorial

Prof. Wei Zhang

Southern University of Science and Technology

zhangw3@sustech.edu.cn
https://www.wzhanglab.site/

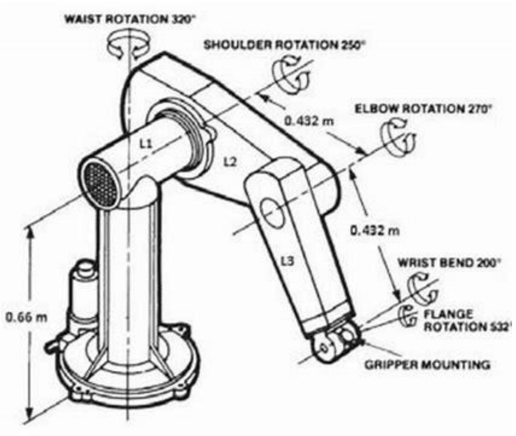
Outline

- **Short introduction to Simulation**
- Introduction to Mujoco
- Python Example

- **What is Simulation?**
  - Real-world physics are often described by functions, ODE or PDE

  - All simulators essentially solve the ODEs and/or PDEs corresponding to a physical process of interest

- **Three pillars of a simulator:**
  1. Constructing the differential equations/models
  2. Solving differential equations
  3. Visualization of the simulation results



Dynamics equations of Puma 560 Arm

$$I_2 = I_{zz2} + m_2 * (r_{z2}^2 + r_{y2}^2) + (m_3 + m_4 + m_5 + m_6) * a_2^2 ;$$

$$I_3 = -I_{xx2} + I_{yy2} + (m_3 + m_4 + m_5 + m_6) * a_2^2 \quad m_2 * r_{z2}^2 - m_2 * r_{y2}^2 ;$$

$$I_4 = m_2 * r_{z2} * (d_2 + r_{z2}) + m_3 * a_2 * r_{z3} + (m_3 + m_4 + m_5 + m_6) * a_2 * (d_2 + d_3) ;$$

$$I_5 = -m_3 * a_2 * r_{y3} + (m_4 + m_5 + m_6) * a_2 * d_4 + m_4 * a_2 * r_{z4} ;$$

$$I_6 = I_{zz3} + m_3 * r_{y3}^2 + m_4 * a_3^2 + m_4 * (d_4 + r_{z4})^2 + I_{yy4} + m_5 * a_3^2 + m_5 * d_4^2 + I_{zz5} + m_6 * a_3^2 + m_6 * d_4^2 + m_6 * r_{z6}^2 + I_{xx6} ;$$

$$I_7 = m_3 * r_{y3}^2 + I_{zz3} - I_{yy3} + m_4 * r_{z4}^2 + 2 * m_4 * d_4 * r_{z4} + (m_4 + m_5 + m_6) * (d_4^2 - a_3^2) + I_{yy4} - I_{zz4} + I_{zz5} - I_{yy5} + m_6 * r_{z6}^2 - I_{zz6} + I_{xx6} ;$$

$$I_8 = -m_4 * (d_2 + d_3) * (d_4 + r_{z4}) - (m_5 + m_6) * (d_2 + d_3) * d_4 \quad m_3 * r_{y3} * r_{z3} + m_3 * (d_2 + d_3) * r_{y3} ;$$

$$I_9 = m_2 * r_{y2} * (d_2 + r_{z2}) ;$$

$$I_{10} = 2 * m_4 * a_3 * r_{z4} + 2 * (m_4 + m_5 + m_6) * a_3 * d_4 ;$$

$$I_{11} = -2 * m_2 * r_{z2} * r_{y2} ;$$

$$I_{12} = (m_4 + m_5 + m_6) * a_2 * a_3 ;$$

$$I_{13} = (m_4 + m_5 + m_6) * a_3 * (d_2 + d_3) ;$$

$$I_{14} = I_{zz4} + I_{yy5} + I_{zz6} ;$$

$$I_{15} = m_6 * d_4 * r_{z6} ;$$

$$I_{16} = m_6 * a_2 * r_{z6} ;$$

$$I_{17} = I_{zz5} + I_{xx6} + m_6 * r_{z6}^2 ;$$

$$I_{18} = m_6 * (d_2 + d_3) * r_{z6} ;$$

$$I_{19} = I_{yy4} - I_{zz4} + I_{zz5} - I_{yy5} + m_6 * r_{z6}^2 + I_{xx6} - I_{zz6} ;$$

$$I_{20} = I_{yy5} - I_{zz5} - m_6 * r_{z6}^2 + I_{zz6} - I_{xx6} ;$$

$$I_{21} = I_{zz4} - I_{yy4} + I_{zz5} - I_{zz5} ;$$

$$I_{22} = m_6 * a_3 * r_{z6} ;$$

$$I_{23} = I_{zz6} ;$$

Part II. Gravitional Constants

$$g_1 = -g * ((m_3 + m_4 + m_5 + m_6) * a_2 + m_2 * r_{x2}) ;$$

$$g_2 = g * (m_3 * r_{y3} - (m_4 + m_5 + m_6) * d_4 - m_4 * r_{z4}) ;$$

$$g_3 = g * m_2 * r_{y2} ;$$

$$g_4 = -g * (m_4 + m_5 + m_6) * a_3 ;$$

$$g_5 = -g * m_6 * r_{z6} ;$$

Table A3. Computed Values for the Constants Appearing in the Equations of Forces of Motion.

(Inertial constants have units of kilogram meters-squared)

I_1	$= 1.43$	± 0.05	I_2	$= 1.75$	± 0.07
I_3	$= 1.38$	± 0.05	I_4	$= 6.90\times10^{-1}$	$\pm 0.20\times10^{-1}$
I_5	$= 3.72\times10^{-1}$	$\pm 0.31\times10^{-1}$	I_6	$= 3.33\times10^{-1}$	$\pm 0.16\times10^{-1}$
I_7	$= 2.98\times10^{-1}$	$\pm 0.29\times10^{-1}$	I_8	$= -1.34\times10^{-1}$	$\pm 0.14\times10^{-1}$
I_9	$= 2.38\times10^{-2}$	$\pm 1.20\times10^{-2}$	I_{10}	$= -2.13\times10^{-2}$	$\pm 0.22\times10^{-2}$
I_{11}	$= -1.42\times10^{-2}$	$\pm 0.70\times10^{-2}$	I_{12}	$= -1.10\times10^{-2}$	$\pm 0.11\times10^{-2}$
I_{13}	$= -3.79\times10^{-3}$	$\pm 0.90\times10^{-3}$	I_{14}	$= 1.64\times10^{-3}$	$\pm 0.07\times10^{-3}$
I_{15}	$= 1.25\times10^{-3}$	$\pm 0.30\times10^{-3}$	I_{16}	$= 1.24\times10^{-3}$	$\pm 0.30\times10^{-3}$
I_{17}	$= 6.42\times10^{-4}$	$\pm 3.00\times10^{-4}$	I_{18}	$= 4.31\times10^{-4}$	$\pm 1.30\times10^{-4}$
I_{19}	$= 3.00\times10^{-4}$	$\pm 14.0\times10^{-4}$	I_{20}	$= -2.02\times10^{-4}$	$\pm 8.00\times10^{-4}$
I_{21}	$= -1.00\times10^{-4}$	$\pm 6.00\times10^{-4}$	I_{22}	$= -5.80\times10^{-5}$	$\pm 1.50\times10^{-5}$
I_{23}	$= 4.00\times10^{-5}$	$\pm 2.00\times10^{-5}$			
I_{m1}	$= 1.14$	± 0.27	I_{m2}	$= 4.71$	± 0.54
I_{m3}	$= 8.27\times10^{-1}$	$\pm 0.93\times10^{-1}$	I_{m4}	$= 2.00\times10^{-1}$	$\pm 0.16\times10^{-1}$
I_{m5}	$= 1.79\times10^{-1}$	$\pm 0.14\times10^{-1}$	I_{m6}	$= 1.93\times10^{-1}$	$\pm 0.16\times10^{-1}$

(Gravitational constants have units of newton meters)

g_1	$= -37.2$	± 00.5	g_2	$= -8.44$	± 0.20
g_3	$= 1.02$	± 0.50	g_4	$= 2.49\times10^{-1}$	$\pm 0.25\times10^{-1}$
g_5	$= -2.82\times10^{-2}$	$\pm 0.56\times10^{-2}$			

$$b_{224} = 2 * \{-I_{16} * C3 * S4 * S5 + I_{20} * SC4 * SS5 + I_{21} * SC4 - I_{22} * S4 * S5\} ; \approx -2.48\times10^{-3} * C3 * S4 * S5 .$$

$$b_{225} = 2 * \{-I_{15} * S5 + I_{16} * (C3 * C4 * C5 - S3 * S5) + I_{20} * SS4 * SC5 + I_{22} * C4 * C5\} ; \approx -2.50\times10^{-3} * S5 + 2.48\times10^{-3} * (C3 * C4 * C5 - S3 * S5) .$$

$$b_{226} = 0 . \qquad b_{234} = b_{224} .$$

$$b_{235} = b_{225} . \qquad b_{236} = 0 .$$

$$b_{245} = 2 * \{-I_{15} * S4 * C5 - I_{16} * S3 * S4 * C5\} - I_{17} * S4 + I_{20} * S4 * (1 - 2 * SS5) ; \quad \approx 0 .$$

$$b_{246} = I_{23} * C4 * S5 ; \quad \approx 0 .$$

$$b_{256} = I_{23} * S4 * C5 ; \quad \approx 0 .$$

$$b_{312} = 0 . \qquad b_{313} = 0 .$$

$$b_{314} = 2 * \{-I_{15} * C23 * C4 * S5 + I_{22} * S23 * C4 * S5 + I_{20} * (S23 * (CC5 * CC4 - 0.5) + C23 * C4 * SC5)\} + I_{14} * S23 + I_{19} * S23 * (1 - (2 * SS4)) ; \approx -2.50\times10^{-3} * C23 * C4 * S5 + 1.64\times10^{-3} * S23 + 0.30\times10^{-3} * S23 * (1 - 2 * SS4) .$$

$$b_{315} = 2 * (-I_{15} * C23 * S4 * C5 + I_{22} * S23 * S4 * C5) - I_{17} * C23 * S4 + I_{20} * S4 * (C23 * (1 - 2 * SS5) - 2 * S23 * C4 * SC5) ; \approx -2.50\times10^{-3} * C23 * S4 * C5 - 6.42\times10^{-4} * C23 * S4 .$$

$$b_{316} = -b_{136} . \qquad b_{323} = 0 .$$

$$b_{324} = 2 * \{I_{20} * SC4 * SS5 + I_{21} * SC4 - I_{22} * S4 * S5\} ; \approx 0 .$$

$$b_{325} = 2 * \{-I_{15} * S5 + I_{20} * SS4 * SC5 + I_{22} * C4 * C5\} ; \approx -2.50\times10^{-3} * S5 .$$

$$b_{326} = 0 . \qquad b_{334} = b_{324} .$$

$$b_{335} = b_{325} . \qquad b_{336} = 0 .$$

$$b_{345} = -I_{15} * 2 * S4 * C5 - I_{17} * S4 + I_{20} * S4 * (1 - 2 * SS5) ; \approx -2.50\times10^{-3} * S4 * C5 .$$

$$b_{346} = b_{246} . \qquad b_{356} = b_{256} . \qquad b_{412} = -b_{214} .$$

$$b_{413} = -b_{314} . \qquad b_{414} = 0 .$$

$$b_{415} = -I_{20} * (S23 * C4 * (1 - 2 * SS5) + 2 * C23 * SC5) - I_{17} * S23 * C4 ; \approx -6.42\times10^{-4} * S23 * C4 .$$

$$b_{416} = -b_{146} . \qquad b_{423} = -b_{324} . \qquad b_{424} = 0 .$$

$$b_{425} = I_{17} * S4 + I_{20} * S4 * (1 - 2 * SS5) ; \approx 6.42\times10^{-4} * S4 .$$

$$b_{426} = -b_{246} . \qquad b_{434} = 0 .$$

$$b_{435} = b_{425} . \qquad b_{436} = -b_{346} .$$

$$b_{445} = -I_{20} * 2 * SC5 ; \quad \approx 0 .$$

$$b_{446} = 0 ;$$

$$b_{456} = -I_{23} * S5 ; \quad \approx 0 .$$

$$b_{512} = -b_{215} . \qquad b_{513} = -b_{315} . \qquad b_{514} = -b_{415} .$$

$$b_{515} = 0 . \qquad b_{516} = -b_{156} . \qquad b_{523} = -b_{325} .$$

$$b_{524} = -b_{425} . \qquad b_{525} = 0 . \qquad b_{526} = -b_{256} .$$

$$b_{534} = b_{524} . \qquad b_{535} = 0 . \qquad b_{536} = -b_{356} .$$

$$b_{545} = 0 . \qquad b_{546} = -b_{456} . \qquad b_{556} = 0 .$$

$$b_{612} = b_{126} . \qquad b_{613} = b_{136} . \qquad b_{614} = b_{146} .$$

$$b_{615} = b_{156} . \qquad b_{616} = 0 . \qquad b_{623} = 0 .$$

$$b_{624} = b_{246} . \qquad b_{625} = b_{256} . \qquad b_{626} = 0 .$$

$$b_{634} = b_{624} . \qquad b_{635} = b_{625} . \qquad b_{636} = 0 .$$

$$b_{645} = b_{456} . \qquad b_{646} = 0 . \qquad b_{656} = 0 .$$

Table A6. The expressions for the terms of the centrifugal matrix. (The Abbreviated Expressions have units of kg-m².)

$$c_{11} = 0 .$$

$$c_{12} = +I4 * C2 - I8 * S23 - I9 * S2 + I13 * C23 + I_{15} * S23 * S4 * S5 + I_{16} * C2 * S4 * S5 + I_{18} * (C23 * C4 * S5 + S23 * C5) + I_{19} * C23 * SC4 + I_{20} * S4 * (C23 * C4 * cC5 - S23 * SC5) + I_{22} * C23 * S4 * S5 ; \approx 6.90\times10^{-1} * C2 + 1.34\times10^{-1} * S23 - 2.38\times10^{-2} * S2 .$$

$$c_{13} = 0.5 * b_{123} .$$

$$c_{14} = -I_{15} * S23 * S4 * S5 - I_{16} * C2 * S4 * S5 + I_{18} * S23 * C4 * S5 + I_{20} * S23 * S4 * SC5 - I_{22} * C23 * S4 * S5 ; \quad \approx 0 .$$

$$c_{15} = -I_{15} * S23 * S4 * S5 - I_{16} * C2 * S4 * S5 ; + I_{18} * (S23 * C5 + C23 * C4 * S5) - I_{22} * C23 * S4 * S5 \approx 0 .$$

$$c_{16} = 0 . \qquad c_{21} = -0.5 * b_{112} .$$

$$c_{22} = 0 . \qquad c_{23} = 0.5 * b_{223} .$$

$$c_{24} = -I_{15} * C4 * S5 - I_{16} * S3 * C4 * S5 + I_{20} * C4 * SC5 ; \approx 0 .$$

$$c_{25} = -I_{15} * C4 * S5 + I_{16} * (C3 * C5 - S3 * C4 * S5) + I_{22} * C5 ; \quad \approx 0 .$$

$$c_{26} = 0 . \qquad c_{31} = -0.5 * b_{113} .$$

$$c_{32} = -c_{23} . \qquad c_{33} = 0 .$$

$$c_{34} = -I_{15} * C4 * S5 + I_{20} * C4 * SC5 ; \approx -1.25\times10^{-3} * C4 * S5 .$$

$$c_{35} = -I_{15} * C4 * S5 + I_{22} * C5 ; \quad \approx c_{34} .$$

$$c_{36} = 0 . \qquad c_{41} = -0.5 * b_{114} . \qquad c_{42} = -0.5 * b_{224} .$$

$$c_{43} = 0.5 * b_{423} . \qquad c_{44} = 0 . \qquad c_{45} = 0 .$$

$$c_{46} = 0 . \qquad c_{51} = -0.5 * b_{115} . \qquad c_{52} = -0.5 * b_{225} .$$

$$c_{53} = 0.5 * b_{523} . \qquad c_{54} = -0.5 * b_{445} . \qquad c_{55} = 0 .$$

$$c_{56} = 0 . \qquad c_{61} = 0 . \qquad c_{62} = 0 .$$

$$c_{63} = 0 . \qquad c_{64} = 0 . \qquad c_{65} = 0 .$$

$$c_{66} = 0 .$$

Table A7. Gravity Terms. (The Abbreviated Expressions have units of newton-meters.)

$$g_1 = 0 .$$

$$g_2 = g1 * C2 + g2 * S23 + g3 * S2 + g4 * C23 ; + g5 * (S23 * C5 + C23 * C4 * S5) \approx -37.2 * C2 - 8.4 * S23 + 1.02 * S2 .$$

$$g_3 = g2 * S23 + g4 * C23 + g5 * (S23 * C5 + C23 * C4 * S5) ; \approx -8.4 * S23 + 0.25 * C23 .$$

$$g_4 = -g5 * S23 * S4 * S5 ; \approx 2.8\times10^{-2} * S23 * S4 * S5 .$$

$$g_5 = g5 * (C23 * S5 + S23 * C4 * C5) ; \approx -2.8\times10^{-2} * (C23 * S5 + S23 * C4 * C5) .$$

$$g_6 = 0 .$$

Table A4. The expressions giving the elements of the kinetic energy matrix. (The Abbreviated Expressions have units of kg-m².)

$$a_{11} = I_{m1} + I_1 + I_3 * CC2 + I_7 * SS23 + I_{10} * SC23 + I_{11} * SC2 + I_{20} * (SS5 * (SS23 * (1 + CC4) - 1) - 2 * SC23 * C4 * SC5) + I_{21} * SS23 * CC4 + 2 * \{I_5 * C2 * S23 + I_{12} * C2 * C23 + I_{15} * (SS23 * C5 + SC23 * C4 * S5) + I_{16} * C2 * (S23 * C5 + C23 * C4 * S5) + I_{18} * S4 * S5 + I_{22} * (SC23 * C5 + CC23 * C4 * S5)\} ; \approx 2.57 + 1.38 * CC2 + 0.30 * SS23 + 7.44\times10^{-1} * C2 * S23 .$$

$$a_{12} = I_4 * S2 + I_8 * C23 + I_9 * C2 + I_{13} * S23 - I_{15} * C23 * S4 * S5 + I_{16} * S2 * S4 * S5 + I_{18} * (S23 * C4 * S5 - C23 * C5) + I_{19} * S23 * SC4 + I_{20} * S4 * (S23 * C4 * CC5 + C23 * SC5) + I_{22} * S23 * S4 * S5 ; \approx 6.90\times10^{-1} * S2 - 1.34\times10^{-1} * C23 + 2.38\times10^{-2} * C2 .$$

$$a_{13} = I_8 * C23 + I_{13} * S23 - I_{15} * C23 * S4 * S5 + I_{19} * S23 * SC4 + I_{18} * (S23 * C4 * S5 - C23 * C5) + I_{22} * S23 * S4 * S5 + I_{20} * S4 * (S23 * C4 * CC5 + C23 * SC5) ; \approx -1.34\times10^{-1} * C23 + -3.97\times10^{-3} * S23 .$$

$$a_{14} = I_{14} * C23 + I_{15} * S23 * C4 * S5 + I_{16} * C2 * C4 * S5 + I_{18} * C23 * S4 * S5 - I_{20} * (S23 * C4 * SC5 + C23 * SS5) + I_{22} * C23 * C4 * S5 ; \quad \approx 0 .$$

$$a_{15} = I_{15} * S23 * S4 * C5 + I_{16} * C2 * S4 * C5 + I_{17} * S23 * S4 + I_{18} * (S23 * S5 - C23 * C4 * C5) + I_{22} * C23 * S4 * C5 ; \approx 0 .$$

$$a_{16} = I_{23} * (C23 * C5 - S23 * C4 * S5) ; \quad \approx 0 .$$

$$a_{22} = I_{m2} + I_2 + I_6 + I_{20} * SS4 * SS5 + I_{21} * SS4 + 2 * \{I_5 * S3 + I_{12} * C3 + I_{15} * C5 + I_{16} * (S3 * C5 + C3 * C4 * S5) + I_{22} * C4 * S5\} ; \approx 6.79 + 7.44\times10^{-1} * S3 .$$

$$a_{23} = I_5 * S3 + I_6 + I_{12} * C3 + I_{16} * (S3 * C5 + C3 * C4 * S5) + I_{20} * SS4 * SS5 + I_{21} * SS4 + 2 * \{I_{15} * C5 + I_{22} * C4 * S5\} ; \approx .333 + 3.72\times10^{-1} * S3 - 1.10\times10^{-2} * C3 .$$

$$a_{24} = -I_{15} * S4 * S5 - I_{16} * S3 * S4 * S5 + I_{20} * S4 * SC5 ; \approx 0 .$$

$$a_{25} = I_{15} * C4 * C5 + I_{16} * (C3 * S5 + S3 * C4 * C5) + I_{17} * C4 + I_{22} * S5 ; \quad \approx 0 .$$

$$a_{26} = I_{23} * S4 * S5 ; \quad \approx 0 .$$

$$a_{33} = I_{m3} + I_6 + I_{20} * SS4 * SS5 + I_{21} * SS4 + 2 * \{I_{15} * C5 + I_{22} * C4 * S5\} ; \quad \approx 1.16 .$$

$$a_{34} = -I_{15} * S4 * S5 + I_{20} * S4 * SC5 ; \approx -1.25\times10^{-3} * S4 * S5 .$$

$$a_{35} = I_{15} * C4 * C5 + I_{17} * C4 + I_{22} * S5 ; \approx 1.25\times10^{-3} * C4 * C5 .$$

$$a_{36} = I_{23} * S4 * S5 ; \quad \approx 0 .$$

$$a_{44} = I_{m4} + I_{14} - I_{20} * SS5 ; \quad \approx 0.20 .$$

$$a_{45} = 0.$$

$$a_{46} = I_{23} * C5 ; \quad \approx 0 .$$

$$a_{55} = I_{m5} + I_{17} ; \quad \approx 0.18 .$$

$$a_{56} = 0.$$

$$a_{66} = I_{m6} + I_{23} ; \quad \approx 0.19 .$$

Table A5. The expressions giving the elements of the Coriolis matrix. (The Abbreviated Expressions have units of kg-m².)

$$b_{112} = 2 * \{-I_3 * SC2 + I_5 * C223 + I_7 * SC23 - I_{12} * S223 + I_{15} * (2 * SC23 * C5 + (1 - 2 * SS23) * C4 * S5) + I_{16} * (C223 * C5 - S223 * C4 * S5) + I_{21} * SC23 * CC4 + I_{20} * ((1 + CC4) * SC23 * SS5 - (1 - 2 * SS23) * C4 * SC5) + I_{22} * ((1 - 2 * SS23) * C5 - 2 * SC23 * C4 * S5)\} + I_{10} * (1 - 2 * SS23) + I_{11} * (1 - 2 * SS2) ; \approx -2.76 * SC2 + 7.44\times10^{-1} * C223 + 0.60 * SC23 - 2.13\times10^{-2} * (1 - 2 * SS23) .$$

$$b_{113} = 2 * \{I_5 * C2 * C23 + I_7 * SC23 - I_{12} * C2 * S23 + I_{15} * (2 * SC23 * C5 + (1 - 2 * SS23) * C4 * S5) + I_{16} * C2 * (C23 * C5 - S23 * C4 * S5) + I_{21} * SC23 * CC4 + I_{20} * ((1 + CC4) * SC23 * SS5 - (1 - 2 * SS23) * C4 * SC5) + I_{22} * ((1 - 2 * SS23) * C5 - 2 * SC23 * C4 * S5)\} + I_{10} * (1 - 2 * SS23) ; \approx 7.44\times10^{-1} * C2 * C23 + 0.60 * SC23 + 2.20\times10^{-2} * C2 * S23 - 2.13\times10^{-2} * (1 - 2 * SS23) .$$

$$b_{114} = 2 * \{-I_{15} * SC23 * S4 * S5 - I_{16} * C2 * C23 * S4 * S5 + I_{18} * C4 * S5 - I_{20} * (SS23 * SS5 * SC4 - SC23 * S4 * SC5) - I_{22} * CC23 * S4 * S5 - I_{21} * SS23 * SC4\} ; \approx -2.50\times10^{-3} * SC23 * S4 * S5 + 8.60\times10^{-4} * C4 * S5 - 2.48\times10^{-3} * C2 * C23 * S4 * S5 .$$

$$b_{115} = 2 * \{I_{20} * (SC5 * (CC4 * (1 - CC23) - CC23) - SC23 * C4 * (1 - 2 * SS5)) - I_{15} * (SS23 * S5 - SC23 * C4 * C5) - I_{16} * C2 * (S23 * S5 - C23 * C4 * C5) + I_{18} * S4 * C5 + I_{22} * (CC23 * C4 * C5 - SC23 * S5)\} ; \approx -2.50\times10^{-3} * (SS23 * S5 - SC23 * C4 * C5) - 2.48\times10^{-3} * C2 * (S23 * S5 - C23 * C4 * C5) + 8.60\times10^{-4} * S4 * C5.$$

$$b_{116} = 0 .$$

$$b_{123} = 2 * \{-I_8 * S23 + I_{13} * C23 + I_{15} * S23 * S4 * S5 + I_{18} * (C23 * C4 * S5 + S23 * C5) + I_{19} * C23 * SC4 + I_{20} * S4 * (C23 * C4 * CC5 - S23 * SC5) + I_{22} * C23 * S4 * S5\} ; \approx 2.67\times10^{-1} * S23 - 7.58\times10^{-3} * C23 .$$

$$b_{124} = -I_{18} * 2 * S23 * S4 * S5 + I_{19} * S23 * (1 - (2 * SS4)) + I_{20} * S23 * (1 - 2 * SS4 * CC5) - I_{14} * S23 ; \quad \approx 0 .$$

$$b_{125} = I_{17} * C23 * S4 + I_{18} * 2 * (S23 * C4 * C5 + C23 * S5) + I_{20} * S4 * (C23 * (1 - 2 * SS5) - S23 * C4 * 2 * SC5) ; \approx 0 .$$

$$b_{126} = -I_{23} * (S23 * C5 + C23 * C4 * S5) ; \quad \approx 0 .$$

$$b_{134} = b_{124} . \qquad b_{135} = b_{125} . \qquad b_{136} = b_{126} .$$

$$b_{145} = 2 * \{I_{15} * S23 * C4 * C5 + I_{16} * C2 * C4 * C5 + I_{18} * C23 * S4 * C5 + I_{22} * C23 * C4 * C5\} + I_{17} * S23 * C4 - I_{20} * (S23 * C4 * (1 - 2 * SS5) + 2 * C23 * SC5) ; \approx 0 .$$

$$b_{146} = I_{23} * S23 * S4 * S5 ; \quad \approx 0 .$$

$$b_{156} = -I_{23} * (C23 * S5 + S23 * C4 * C5) ; \quad \approx 0 .$$

$$b_{212} = 0 . \qquad b_{213} = 0 .$$

$$b_{214} = I_{14} * S23 + I_{19} * S23 * (1 - (2 * SS4)) + 2 * \{-I_{15} * C23 * C4 * S5 + I_{16} * S2 * C4 * S5 + I_{20} * (S23 * (CC5 * CC4 - 0.5) + C23 * C4 * SC5) + I_{22} * S23 * C4 * S5\} ; \approx 1.64\times10^{-3} * S23 - 2.50\times10^{-3} * C23 * C4 * S5 + 2.48\times10^{-3} * S2 * C4 * S5 + 0.30\times10^{-3} * S23 * (1 - (2 * SS4)) .$$

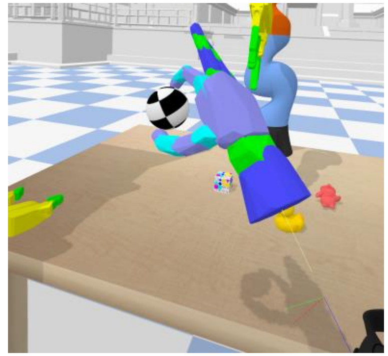
$$b_{215} = 2 * \{-I_{15} * C23 * S4 * C5 + I_{22} * S23 * S4 * C5 + I_{16} * S2 * S4 * C5\} - I_{17} * C23 * S4 + I_{20} * (C23 * S4 * (1 - 2 * SS5) - 2 * S23 * SC4 * SC5) ; \approx -2.50\times10^{-3} * C23 * S4 * C5 + 2.48\times10^{-3} * S2 * S4 * C5 - 6.42\times10^{-4} * C23 * S4 .$$

$$b_{216} = -b_{126} .$$

$$b_{223} = 2 * \{-I_{12} * S3 + I_5 * C3 + I_{16} * (C3 * C5 - S3 * C4 * S5)\} ; \approx 2.20\times10^{-2} * S3 + 7.44\times10^{-1} * C3 .$$

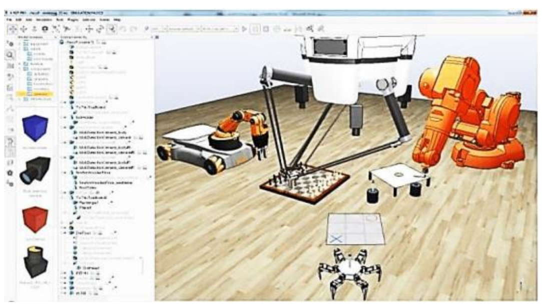
▪ Popular simulators in robotics

Mujoco (Roboti LLC)



PyBullet (open source)

Isaac Sim (NVIDIA)

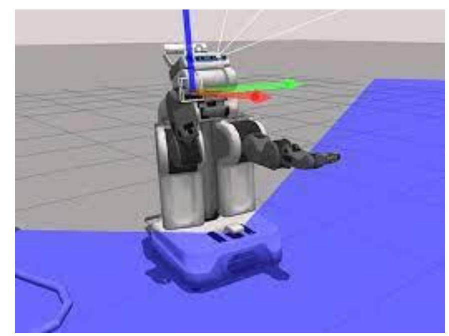


V-REP (CoppeliaSim)

DRAKE

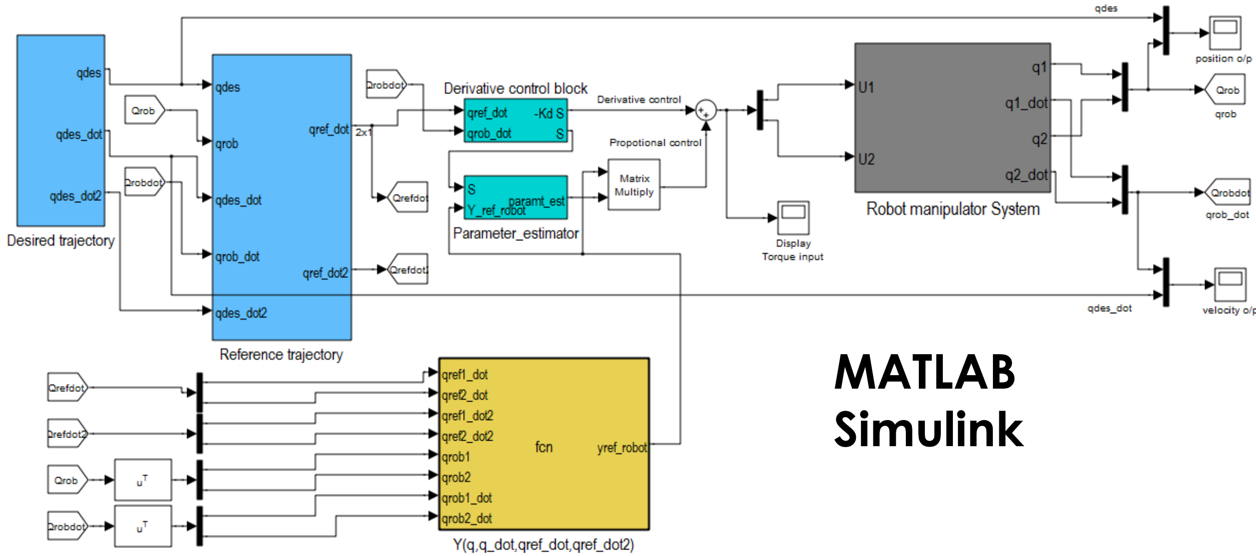
A PLANNING, CONTROL, AND ANALYSIS TOOLBOX FOR NONLINEAR DYNAMICAL SYSTEMS

Drake(Open source）



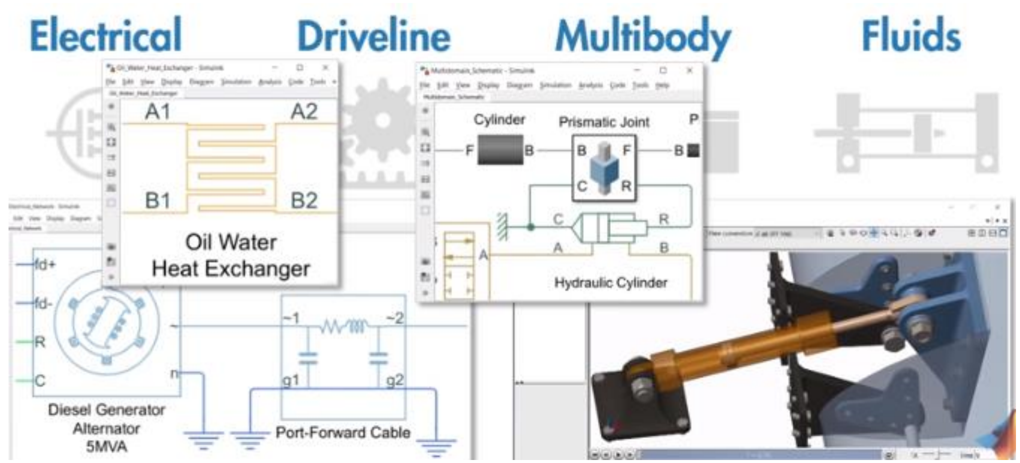
Gazebo

- Popular simulators for control systems



MATLAB Simulink

MATLAB Simscape



Outline

- Short introduction to Simulation
- **Introduction to Mujoco**
- Python Example

Mujoco

- **High-Performance Physics Engine**: MuJoCo offers highly accurate simulations of complex physical interactions, ideal for robotics research
- **Fast Real-Time Simulations**: Its optimization allows for real-time performance, making it suitable for reinforcement learning applications
- **Advanced Contact Dynamics**: MuJoCo handles contact and friction with soft constraints, providing realistic interactions in dynamic environments.
- **Drawbacks:**
    - Lacks of detailed sensor models
    - Struggle with large or highly diverse environments

How to define a robot control system?

```
xml4="""<mujoco model="3R_robot">
    <compiler angle="degree"/>
    <asset>
        <texture name="grid" type="2d" builtin="checker" rgb1=".1 .2 .3"
        rgb2=".2 .3 .4" width="300" height="300" mark="none"/>
        <material name="grid" texture="grid" texrepeat="6 6" texuniform="true" reflectance=".2"/>
    </asset>
    <default>
        <joint type="hinge" axis="0 0 1" limited="true"/>
        <geom type="cylinder" size=".025 .1" />
    </default>

    <worldbody>
        <light diffuse=".5 .5 .5" pos="0 0 3" dir="0 0 -1"/>
        <geom type="plane" size="1 1 0.1" material="grid"/>

        <body name="BaseLink" pos="0 0 0.1">
            <geom type="cylinder" pos="0 0 0" size=".025 .1" />
            <body name="link1" pos="0 0.1 0.125" euler="-90 0 0">
                <joint name="joint1" pos="0 0 -0.1" range="-90 90" axis ="0 1 0"/>
                <geom pos="0 0 0" rgba=".6 .2 .2 1"/>
                <site name="torque_site" pos="0 0.2 0"/>
                <body name="link2" pos="0 0 0.2">
                    <joint name="joint2" pos="0 0 -0.1" range="-90 90" axis="0 1 0"/>
                    <geom rgba=".2 .6 1 1"/>
                    <site name="end_effector" pos="0 0 0.1" size="0.01"/>
                </body>
            </body>
        </body>
    </worldbody>
</mujoco>"""
```

```
import mujoco
m = mujoco.MjModel.from_xml_string(xml4)
m = mujoco.MjModel.from_xml_path('***.xml')
d = mujoco.MjData(m)
```

How to define a robot control system?

- We focus on **rigid body** system: Multiple rigid bodies interconnected through joints.

How to define **a rigid body**?

1. Where is it?

2. How does it look?

3. How does it connect to others?

4. Its physical properties?

▪ How to define a rigid body?

1. Where is it?

- body element:
  - <name>: optional
  - <pos>:
  - <euler>, or <quat> or <axisangle>: specify frame orientation relative to parent frame, optional (default orientation matrix is identity)

```
<worldbody>
    <body name="BaseLink" pos="0 0 0.1">
        <body name="link1" pos="0 0.1 0.125" euler="-90 0 0">
            <body name="link2" pos="0 0 0.2">
            </body>
        </body>
    </body>
</worldbody>
```

▪ How to define a rigid body?

2. How does it look?

geom: sub-element of body

- <name> (optional), position <pos>, orientation
- <type>: sphere (default), plane, capsule, ellipsoid, cylinder, box, mesh, sdf

Type	Number	Description
plane	3	X half-size; Y half-size; spacing between square grid lines for rendering. If either the X or Y half-size is 0, the plane is rendered as infinite in the dimension(s) with 0 size.
hfield	0	The geom sizes are ignored and the height field sizes are used instead.
sphere	1	Radius of the sphere.
capsule	1 or 2	Radius of the capsule; half-length of the cylinder part when not using the fromto specification.
ellipsoid	3	X radius; Y radius; Z radius.
cylinder	1 or 2	Radius of the cylinder; half-length of the cylinder when not using the fromto specification.
box	3	X half-size; Y half-size; Z half-size.
mesh	0	The geom sizes are ignored and the mesh sizes are used instead.

- <fromto>:
- <material>:
- <rgba>:
- <mass>: optional
- <density>: default "1000": density of water in SI unit

▪ How to define a rigid body?

3. How does it connect to others?

- joint: sub-element of body

▪ How to define a rigid body?

4. Its physical properties?

Type 1 (default): infer from geom attached to the body

Type 2: inertia sub-element

- <pos>: position of inertial frame.
- <orientation>: of the inertial frame
- <mass>: positive number required
- <diaginertia> (real(3)): diagonal entries of the inertial matrix;
- <fullinertia>: real(6): Full inertia matrix M: M(1,1), M(2,2), M(3,3), M(1,2), M(1,3), M(2,3).

```
<mujoco>
  <worldbody>
    <body name="arm" pos="0 0 0">
      <!-- Define the mass and inertia of this body -->
      <inertial pos="0 0 0" mass="1.0" diaginertia="0.01 0.01 0.01"/>

      <!-- Additional parts of the body, like joints or geometric shapes
      <geom type="capsule" size="0.05 0.2" rgba="0.8 0.3 0.3 1" />
      <joint type="hinge" axis="0 1 0" />
    </body>
  </worldbody>
</mujoco>
```

▪ Assets

Assets are not in themselves model elements. Model elements can reference them. One asset can be referenced by multiple model elements.

- **asset/mesh:** MuJoCo works with triangulated meshes. They can be loaded from binary STL files, OBJ files or MSH files.

- **asset/material:** It can be referenced from skins, geoms, sites and tendons to set their appearance. Materials are useful for adjusting appearance properties beyond color.

```xml
<mujoco>
  <asset>
    <texture name="grid" type="2d" builtin="checker" rgb1=".1 .2 .3"
     rgb2=".2 .3 .4" width="300" height="300" mark="none"/>
    <material name="grid" texture="grid" texrepeat="6 6"
     texuniform="true" reflectance=".2"/>
     <material name="wall" rgba='.5 .5 .5 1'/>
  </asset>
  <default>
    <geom type="box" size=".05 .05 .05" />
    <joint type="free"/>
  </default>

  <worldbody>
    <light name="light" pos="-.2 0 1"/>
    <geom name="ground" type="plane" size="10 10 10" material="grid"
     zaxis="-.3 0 1" friction=".5"/>
    <camera name="y" pos="-.1 -.6 .3" xyaxes="1 0 0 0 1 2"/>
    <body pos="0 1 .3">
      <joint/>
      <geom friction="0.3"/>
    </body>
    <body pos="0 0 .3">
      <joint/>
      <geom friction="1"/>
    </body>
  </worldbody>
```

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- **Python Example**

- Summary