SDM5008 Advanced Control for Robotics

Lecture 7: Rigid Body Dynamics

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Outline

- Spatial Acceleration
- Spatial Force (Wrench)
- Spatial Momentum

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Spatial Force (Wrench)

- Consider a rigid body with many forces on it and fix an arbitrary point O in space

- The net effect of these forces can be expressed as
  - A force f, acting along a line passing through O

  - A moment n_O about point O

- **Spatial Force (Wrench):** is given by the 6D vector

$$\mathcal{F} = \begin{bmatrix} n_O \\ f \end{bmatrix}$$

Spatial Force in Plücker Coordinate Systems

- Given a frame {A}, the Plücker coordinate of a spatial force $\mathcal{F}$ is given by

$$^{A}\mathcal{F} = \begin{bmatrix} ^{A}n_{o_A} \\ ^{A}f \end{bmatrix}$$

- Coordinate transform: $^{A}\mathcal{F} = {}^{A}X_B^{*}\,{}^{B}\mathcal{F}$ where $^{A}X_B^{*} = {}^{B}X_A{}^{T}$

Wrench-Twist Pair and Power

- Recall that for a point mass with linear velocity v and linear force f. Then we know that the power (instantaneous work done by f) is given by $f \cdot v = f^T v$

- This relation can be generalized to spatial force (i.e. wrench) and spatial velocity (i.e. twist)

- Suppose a rigid body has a twist ${}^A\mathcal{V} = ({}^A\omega, {}^A v_{o_A})$ and a wrench ${}^A\mathcal{F} = ({}^A n_{o_A}, {}^A f)$ acts on the body. Then the power is simply

$$P = \left({}^A\mathcal{V}\right)^T \ {}^A\mathcal{F}$$

Joint Torque

- Consider a link attached to a 1-dof joint (e.g. revolute or prismatic). Let $\hat{\mathcal{S}}$ be the screw axis of the joint. The velocity of the link induced by joint motion is given by:$\mathcal{V} = \hat{\mathcal{S}}\dot{\theta}$

- $\mathcal{F}$ be the wrench provided by the joint. Then the power produced by the joint is

$$P = \mathcal{V}^T \mathcal{F} = (\hat{\mathcal{S}}^T \mathcal{F})\dot{\theta} \triangleq \tau\dot{\theta}$$

- $\tau = \hat{\mathcal{S}}^T \mathcal{F} = \mathcal{F}^T \hat{\mathcal{S}}$ is the projection of the wrench onto the screw axis, i.e. the effective part of the wrench.

- Often times, τ is referred to as joint "torque" or generalized force

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Rotational Inertia (1/2)

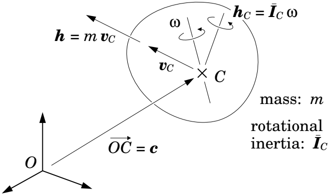
- Recall momentum for point mass:

Rotational Inertia (2/2)

- Rotational Inertia: $\bar{I} = \int_V \rho(r)[r][r]^T dr$
  - $\rho(\cdot)$ is the density function of the body
  - $\bar{I}$ depends on coordinate system
  - It is a constant matrix if the origin coincides with CoM

Spatial Momentum

- Consider a rigid body with spatial velocity $\mathcal{V}_C = (\omega, v_C)$ expressed at the center of mass C
  - Linear momentum:
  - Angular momentum about CoM:
  - Angular momentum about a point O:
- Spatial Momentum:



Change Reference Frame for Momentum

- Spatial momentum transforms in the same way as spatial forces:

$$^{A}h = {}^{A}X_{C}^{*}\,{}^{C}h$$

Spatial Inertia

- Inertia of a rigid body defines linear relationship between velocity and momentum.

- Spacial inertia $\mathcal{I}$ is the one such that

$$h = \mathcal{I}\mathcal{V}$$

- Let {C} be a frame whose origin coincide with CoM. Then

$$^{c}\mathcal{I} = \begin{bmatrix} ^{c}\bar{I}_c & 0 \\ 0 & mI_3 \end{bmatrix}$$

Spatial Inertia

- Spatial inertia wrt another frame {A}:

$$^{A}\mathcal{I} = {}^{A}X_{C}^{*}\, {}^{C}\mathcal{I}\, {}^{C}X_{A}$$

- Special case: ${}^{A}R_{C} = I_3$

More Discussions

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